

# ARBITRAGE RISK AND MARKET EFFICIENCY— APPLICATIONS TO SECURITIES CLASS ACTIONS

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## INTRODUCTION

Market efficiency has been widely studied in the field of finance for decades, as it provides an investor with a sense of how well the price signal works at conveying all available information, and thus informs an investor of the necessity to acquire additional information about the firm issuing the security.<sup>1</sup> Market efficiency has gained acceptance within the

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1. For some of the myriad academic research on market efficiency and its tests, see, e.g., Eugene Fama, *Efficient Capital Markets: A Review of Theory and Empirical Work*, 25 J. FIN. 383 (1970); Bradford Cornell, *Spot Rates, Forward Rates and Exchange Market Efficiency*, 5 J. FIN. ECON. 55 (1977); Michael Brennan & Eduardo Schwartz, *An Equilibrium Model of Bond Pricing and a Test*

court system as a means of facilitating proof in securities fraud litigation. In particular, in the 1988 case of *Basic v. Levinson*, the United States Supreme Court firmly established the fraud-on-the-market theory as a means for securities fraud plaintiffs to satisfy the legal element that they had relied upon a material misrepresentation or omission in purchasing or selling a security. While the courts have recently reexamined whether the legal sector's use of the efficient market theory is justified, it remains firmly entrenched in judicial analysis. Thus detailed economic analysis of market efficiency will continue to play a significant role in securities cases.

Because reliance is a required element of securities fraud cases and because class action procedures generally require that plaintiffs show that reliance can be proven on a class-wide basis, courts most frequently assess market efficiency at the class certification stage of securities fraud cases—the point at which the court determines if the plaintiffs' claims are best tried individually or whether numerous plaintiffs can collectively pursue essentially the same claim against the defendant at the same time.<sup>2</sup> Trial courts thus devote significant time and energy to determinations about market efficiency in deciding whether to certify a case for class action treatment.

In *Cammer v. Bloom* (D. N.J. 1989), the federal district court enumerated several factors for determining market efficiency of the securities in question: (1) the average weekly trading volume, (2) the number of security analysts following and reporting on the security, (3) the extent to which market

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*of Market Efficiency*, 17 J. FIN. & QUANTITATIVE ANALYSIS 301 (1982); Gerald Dwyer & Myles Wallace, *Cointegration and Market Efficiency*, 11 J. INT'L MONEY & FIN. 318 (1992); Ronald Gilson & Reinier Kraakman, *The Mechanisms of Market Efficiency*, 100 VA L. REV. 313 (1984); Michael Jensen, *Some Anomalous Evidence Regarding Market Efficiency*, 6 J. FIN. ECON. 95 (1978); S.P. Kothari, *Capital Markets Research in Accounting*, 31 J. ECON. & ACCT. 105 (2001); Tim Loughran & Jay Ritter, *Uniformly Least Powerful Tests of Market Efficiency*, 55 J. FIN. ECON. 361 (2000); "Efficient Market Hypothesis." NEW PALGRAVE DICTIONARY OF MONEY AND FINANCE 739–42 (1st ed. 1992); Burton Malkiel, *The Efficient Market Hypothesis and Its Critics*, 17 J. ECON. PERP. 59 (2003); Rafael Porta, Josef Lakonishok, Andrei Shleifer, & Robert Vishny, *Good News for Value Stocks: Further Evidence on Market Efficiency*, 52 J. FIN. 859 (1997); Richard Roll, *A Simple Implicit Measure of the Effective Bid-Ask Spread in an Efficient Market*, 39 J. FIN. 1127 (1984); Paul Samuelson, *An Enjoyable Life Puzzling Over Modern Finance Theory*, 1 ANN. REV. FIN. ECON. 19 (2009); Robert Shiller, *The Use of Volatility Measures in Assessing Market Efficiency*, 36 J. FIN. 291 (1981).

2. See FED. R. CIV. P. 23; *Halliburton Co. v. Erica P. John Fund, Inc.*, \_\_\_ U.S. \_\_\_, \_\_\_, 134 S. Ct. 2398, 2407–08, 2412, 2415–16 (2014).

makers traded the security, (4) the issuer's eligibility to file an SEC registration Form S-3, and (5) the cause-and-effect relationship between material disclosures and changes in the security's price.<sup>3</sup> These "*Cammer* factors" have been adopted by a number of courts, while still other courts have added other factors.<sup>4</sup> For instance, one court considered the company's market capitalization and the size of the public float for the security, while another considered the ability to sell short the security and the level of autocorrelation between the security's prices.<sup>5</sup>

From finance theory, the market for a security is said to be "semistrong form efficient" if the price of the security reflects all publicly available information. Prices of securities reflect, albeit *to varying extents*, all publicly available information; therefore, markets for securities are semistrong form efficient *in varying degrees*. Much research has also been done to determine the mechanisms by which the pricing signal operates, and it is widely understood that correction of mispricing of a stock primarily occurs through arbitrage activity.<sup>6</sup>

Since arbitrage is not a cost-free activity, and because frictions remain, whether in the form of transaction costs, idiosyncratic risk, or other costs and risks associated with trading securities, pricing anomalies may persist.<sup>7</sup> As a result, everything else remaining the same, financial economics tells us that the market for a stock with a higher arbitrage cost will be less efficient—i.e., a stock's market efficiency is negatively related to its arbitrage risk.<sup>8</sup> Thus, we refer to arbitrage risk

3. 711 F. Supp. 1264, 1286–87.

4. See *In re DVI, Inc. Sec. Litig.*, 639 F.3d 623, 633 n.14 (3d Cir. 2011); *Teamsters Local 445 Freight Div. Pension, Fund v. Bombardier, Inc.*, 546 F.3d 196, 204–05 n. 11 (2d Cir. 2008); *In re Xcelera.com Sec. Litig.*, 430 F.3d 503, 508 (1st Cir. 2005); *Unger v. Amedisys Inc.*, 401 F.3d 316, 323 (5th Cir. 2005); *Gariety v. Grant Thornton, LLP*, 368 F.3d 356, 368 (4th Cir. 2004); *Binder v. Gillespie*, 184 F.3d 1059, 1064–65 (9th Cir. 1999).

5. See *Krogman v. Sterritt*, 202 F.R.D. 467, 478 (N.D. Tex. 2001); *In re Polymedica Corp. Sec. Litig.*, 432 F.3d 1, 18 n. 21 (1st Cir. 2005).

6. See, e.g., LARRY HARRIS, *TRADING & EXCHANGES: MARKET MICROSTRUCTURE FOR PRACTITIONERS* Ch. 10 & Ch. 17 (2003); Jeffrey Pontiff, *Costly Arbitrage and the Myth of Idiosyncratic Risk*, 42 J. ACCT. & ECON. 35 (2006).

7. See, e.g., Jeffrey Pontiff, *Costly arbitrage and the myth of idiosyncratic risk*, J. ACCT. & ECON. (2006).

8. This implies that Market Efficiency Percentile – 1 = 100 – Arbitrage Risk Percentile. For example, if a stock is at the 25th percentile for arbitrage risk, then the stock is at the 76th percentile for market efficiency.

as a negative proxy for market efficiency. We discuss this in detail in Part II.

Consider an arbitrageur whose information suggests that a stock is underpriced. The arbitrageur will then “go long” on that stock (buy and hold the stock) in order to obtain arbitrage profits by selling the stock at a later date. However, the arbitrageur will also manage the risk of holding the stock by hedging. As a result of our interviews with traders “in the trenches,” we model the arbitrageur as choosing the optimal hedge stocks and the optimal hedge ratios. The risk of this optimal arbitrage portfolio is the arbitrage risk of the stock, our negative proxy for market efficiency. We discuss these calculations in detail.

We provide a methodology that can calculate the market efficiency percentile of a stock over the relevant period, based on the data for a comparable measurement period.<sup>9</sup> For example, in *Lefkoe, et al. v. Jos. A. Bank Clothiers, Inc.*, where the class period was January 5, 2006 to June 7, 2006, we used August 1, 2005 to January 4, 2006, as the measurement period.

If it is not possible (or desirable) to use a different measurement period—e.g., if the period of interest immediately follows an initial public offering (IPO)—then we can do the calculations with the measurement period as the relevant period, and we call this the *ex post* arbitrage risk of the security for the relevant period. For example, in a recent securities class action filed against Groupon, Inc., the class period was defined as November 4, 2011 to March 30, 2012. Since the class period immediately follows the IPO, we do not have trading data from a prior period to use as the measurement period.

For this paper, we focus on *ex ante* (baseline) arbitrage risk, but we do sensitivity analyses with *ex post* arbitrage risk as another negative proxy for market efficiency. We apply this methodology to calculate, on a yearly basis, the arbitrage risk for each U.S. exchange-listed common stock from 1988 to 2010 (subject to certain restrictions). We also perform a regression analysis of arbitrage risk (as a negative proxy of market efficiency) on the factors identified by courts in securities class actions. These results are summarized in Table 1.<sup>10</sup>

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9. We interpret comparability to mean a time interval that is proximate in location and length.

10. We detail all the variables in Section 4. We use 5% as our level of

<b>Factor</b>	Relation with Market Efficiency	Significance at 5% Level	Consistency with “Conventional Wisdom”
<i>Cammer v. Bloom</i>			
Turnover	<b>Negative</b>	Significant	<b>Inconsistent</b>
Number of Analysts	<b>Negative</b>	Ambiguous	—
Number of Nasdaq Market Makers	<b>Negative</b>	Significant	<b>Inconsistent</b>
<i>Unger v. Amidesys</i>			
Market Capitalization	<b>Positive</b>	Significant	<b>Consistent</b>
Bid-Ask Spread	<b>Negative</b>	Significant	<b>Consistent</b>
Public Float Ratio	<b>Positive</b>	Ambiguous	—
<i>Other</i>			
Institutional Ownership Ratio	<b>Positive</b>	Significant	<b>Consistent</b>
Serial Correlation	<b>Positive</b>	Significant	<b>Inconsistent</b>
Explanatory Power	<b>Positive</b>	Significant	—
Inclusion in Dow Jones Index	<b>Negative</b>	Significant	—

We checked the sensitivity of these results through a number of additional analyses. For one set, we replaced turnover with logarithm of volume (or logarithm of dollar

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significance. If the significance results are different under homoscedasticity and under heteroscedasticity-robustness, we refer to the significance as ambiguous; Halbert White, *A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity*, 48 *ECONOMETRICA* 817 (1980).

volume) but removed market capitalization from the list of factors, reflecting the fact that, *ceteris paribus*, the volume for a stock with higher market capitalization will be higher. For this set, we found that the results were the same as in Table 1, except that market efficiency was positively and significantly affected by number of analysts; positively but insignificantly affected by number of market makers (for Nasdaq stocks); positively but ambiguously affected by serial correlation; and positively and significantly affected by inclusion in the Dow Jones Industrial Average (DJIA) (the latter makes sense because in this set, market capitalization is not used as an explanatory factor, whereas it was used as such for the results in Table 1). The second set uses only the *Cammer* factors as explanatory variables. For this set, we found that the results were the same as in Table 1, except that market efficiency is positively but insignificantly affected by logarithm of volume (or logarithm of dollar volume); and positively and significantly affected by number of analysts.

In Part I, we detail the development and application of market efficiency to securities class actions. In Part II, we develop arbitrage risk as a negative proxy for market efficiency. In Part III, we provide regression results that test the various factors believed to determine market efficiency—we also investigate the empirical findings that are apparently inconsistent with “conventional wisdom” and show that the empirical findings are actually consistent with the principles of financial economics. Part IV concludes the paper.

## I. MARKET EFFICIENCY AND SECURITIES CLASS ACTIONS

General acceptance of the relevance of the efficient market hypothesis by the courts was confirmed with the case of *Basic, Inc. v. Levinson*, 485 U.S. 224 (1988), in which the U.S. Supreme Court adopted the fraud-on-the-market theory. But to understand the courts’ use of market efficiency, it is important first to understand what plaintiffs are required to prove in establishing a securities fraud claim.

In a typical claim of securities fraud pursued under section 10(b) of the Securities Exchange Act of 1934, plaintiffs must prove (1) a material misrepresentation or omission by a defendant, (2) scienter, (3) a connection between the misrepresentation or omission and the purchase or sale of a security, (4) reliance upon the misrepresentation or omission,

(5) economic loss, and (6) loss causation.<sup>11</sup> To justify proceeding as a class action, instead of an individual's claim, plaintiffs must also show that (1) the potential class of affected parties is so large that including them all individually is impractical, (2) questions of law or fact are common to all potential class members, (3) the claims of the named representative are typical of the potential class, and (4) the named representative can fairly and adequately protect the interests of the class.<sup>12</sup> Additionally, plaintiffs must establish at least one of the following: (1) that individual actions risk inconsistent rulings, yielding incompatible standards of conduct or risk impairing the rights of potential class members not a part of the lawsuit, or (2) final injunctive or declarative relief is appropriate, respecting the class as a whole, or (3) the questions of law or fact common to the potential class members predominate over any questions affecting only individual members and that a class action is superior to other methods of adjudication.<sup>13</sup> This last requirement, known as the predominance requirement, is frequently used to establish the additional Rule 23(b) standard for class actions.

Until the adoption of the fraud-on-the-market theory in *Basic*, it was difficult for plaintiffs to establish the reliance element of their claim since they likely bought or sold the underlying security without direct knowledge of the alleged misrepresentation or omission. It was even more challenging to establish that the evidence of reliance by all class members was common to each of them, that all class members relied on the same information and to the same degree in making their securities purchases or sales. The fraud-on-the-market theory was designed to address plaintiffs' difficulties in establishing reliance, with the added benefit that it provided a presumption of reliance applicable to all investors of the security in question.

The fraud-on-the-market theory avoids the pitfall facing plaintiffs by providing them with a rebuttable presumption of reliance upon the alleged misrepresentations so long as the market for the underlying security is efficient.<sup>14</sup> The notion is that, in an open and developed securities market, the price of

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11. *Amgen Inc. v. Connecticut Retirement Plans and Trust Funds*, 133 S. Ct. 1184, 1192 (2013).

12. FED. R. CIV. P. 23(a).

13. FED. R. CIV. P. 23(b).

14. *In re DVI, Inc. Sec. Litig.*, 639 F.3d 623, 631 (3d Cir. 2010).

a security is determined by the publicly available information about the underlying company, including the alleged misrepresentation.<sup>15</sup>

Assessment of market efficiency is generally first presented at the class certification stage of securities fraud cases, the point at which the court resolves whether the plaintiffs' claims are best tried individually or whether numerous plaintiffs can collectively pursue essentially the same claim against the defendant at the same time. At the class certification stage, plaintiffs can present evidence that they traded shares in an efficient market, and the court then presumes that investors who traded securities in that market relied on public, material misrepresentations regarding those securities.<sup>16</sup> Defendants can rebut the presumption of reliance by presenting evidence challenging actual reliance or market efficiency. Based on the evidence presented, the court then decides whether or not the matter can legitimately proceed as a class action.

A class certification hearing is not a trial on the merits and is often conducted before full discovery is completed, so plaintiffs do not need to prove each of the claim elements on the merits at the class certification stage. But plaintiffs are required to prove—not simply plead—the Rule 23(a) class action requirements and, most typically, that questions of law or fact common to all class members predominate over any questions affecting only individual members.<sup>17</sup> Over the years, tensions have grown, however, as the proof required to establish the class action requirements now frequently spills over into the merits of the underlying claims themselves. The courts are thus struggling to determine what and how much information must be proven during class certification contests.

Amid two recent and significant 5-4 decisions reversing class certification decisions because plaintiffs failed to prove the requirements of Rule 23, *Wal-Mart Stores, Inc. v. Dukes*, 564 U.S. \_\_\_ (2011) and *Comcast Corp. v. Behrend*, 569 U.S. \_\_\_ (2013), the United States Supreme Court has now issued three other significant decisions regarding securities class actions cases that ultimately continue to support the 1988

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15. *Erica P. John Fund, Inc.*, 131 S. Ct. 2179, 2181 (2011) (quoting *Basic Inc. v. Levinson*, 485 U.S. 224, 246 (1988)).

16. *Amgen Inc. v. Connecticut Retirement Plans and Trust Funds*, 133 S. Ct. 1184, 1192 (2013).

17. FED. R. CIV. P. 23(b)(3).



*Basic* decision even while demonstrating that the fraud-on-the-market theory and the efficient market theory increasingly are coming under harsh attack.

In *Amgen Inc. v. Connecticut Retirement Plans and Trust Funds*, 568 U.S. \_\_\_ (2013), a 6-3 majority decided that the materiality requirement of a securities claim was sufficiently distinct from market efficiency and the public nature of securities claims such that it did not have to be established at the class certification stage. The Court reasoned that whether a misrepresentation was sufficiently material to a stock price was certainly a matter of common proof such that the courts do not need to delve into the merits of this issue during class certification. The Court essentially held that, while the parties are presenting event studies that go to the reliance (and the predominance of the common reliance evidence) to show that a stock price effect exists, plaintiffs need not prove during class certification that the stock price effect was material. Although certainly implicit in Scalia's short dissenting opinion, neither his dissent nor the dissent of Thomas (joined by Scalia and Kennedy) explicitly suggested that the *Basic* decision should be overruled, presumably because that issue was not directly before the Court.

Amgen is consistent with the Court's unanimous decision two years earlier in *Erica P. John Fund, Inc. v. Halliburton Co.*, \_\_\_ U.S. \_\_\_ (2011), which held that plaintiffs need not prove loss causation, that the misrepresentation in question caused the plaintiffs' economic loss, at the class certification stage. The Fifth Circuit Court of Appeals had previously ruled in favor of Halliburton that plaintiffs' proof of loss causation, that company statements "actually caused the stock price to fall and resulted in the losses," was necessary to invoke the *Basic* presumption of reliance.<sup>18</sup> Before the Supreme Court, Halliburton also suggested that insufficient evidence existed as to any price impact, thus suggesting there was nothing to rely upon in order to invoke the *Basic* presumption.<sup>19</sup> The Supreme Court refused to examine the economic evidence and simply concluded that the Court of Appeals erred in conflating loss causation with the reliance element and the *Basic*

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18. *Erica P. John Fund, Inc. v. Halliburton Co.*, 131 S. Ct. 2179, 2184 (2011) (citations omitted).

19. *Id.* at 2186.

presumption of reliance.<sup>20</sup> The Court remanded the matter for reconsideration of the trial court's class certification decision.

Subsequently, the district court granted class certification, which the Fifth Circuit affirmed.<sup>21</sup> Halliburton then appealed to the Supreme Court and presented two issues. First and foremost, the Court addressed whether the *Basic* presumption of liability should be overruled, and thus whether plaintiffs should be required to prove actual reliance, including whether class-wide, common proof of reliance was now required at the class certification stage of litigation.<sup>22</sup> Second, the Court addressed the extent to which evidence of a presumption of reliance could be rebutted by defendants at the class certification stage, recognizing that class certification hearings are not supposed to be trials on the merits but also recognizing that the Court's recent class action decisions place increasing burdens on plaintiffs to prove (as oppose to presume) the class action requirements of Rule 23.<sup>23</sup>

The Supreme Court yet again unanimously vacated the lower court rulings and instructed the trial court to re-examine the evidence on class certification.<sup>24</sup> Five justices, led by Chief Justice Roberts, determined that Halliburton should be given an opportunity to rebut the *Basic* presumption of reliance by presenting evidence of a lack of any price impact.<sup>25</sup> Justices Ginsburg, Breyer and Sotomayor concurred, recognizing that the evidentiary burden of rebutting the *Basic* presumption falls on defendants and thus should not be an additional hurdle for class action plaintiffs.<sup>26</sup> Justices Thomas, Alito and Scalia concurred in the result but suggested that *Basic* should now be overruled, in part because “overwhelming empirical evidence” now suggests that even when markets do incorporate public information, they often fail to do so accurately” and that “[s]cores of ‘efficiency-defying anomalies—such as market swings in the absence of new information and prolonged deviations from underlying asset values—make market

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20. *Id.*

21. *Halliburton Co. v. Erica P. John Fund, Inc.*, \_\_\_ U.S. \_\_\_, 134 S. Ct. 2398, 2406 (2014).

22. *Id.*

23. *Id.* at 2407.

24. *Id.* at 2417.

25. *Id.*

26. *Id.*

efficiency ‘more contestable than ever.’”<sup>27</sup>

Thus, the *Basic* presumption remains a fixture of federal securities litigation even though the judicial system is now amply aware of the debates within finance theory about the extent and usefulness of the efficient market hypothesis. Furthermore, the academic debates themselves will certainly carry over into future class certification analyses as *Halliburton* supports defendants’ efforts to garner evidence and present their own event studies challenging the efficiency of the information signals associated with plaintiffs’ allegations of misrepresentations.

Without doubt, federal district courts will continue to conduct ever more rigorous reviews of market efficiency at the class certification stage of securities lawsuits. The scope and structure of these analyses are necessarily case-by-case, left to the parties and their financial experts to present evidence to the courts, with the courts then making legal determinations about whether the pertinent markets were “efficient enough” to justify the *Basic* presumption of reliance.

As such, we propose to use our methodology here to examine the market efficiency factors that parties have typically presented to the courts and upon which the courts have relied in making their determinations. In addition to the *Cammer* factors referred to earlier, such factors include market capitalization, size of the public float, ability to sell short the security, and level of autocorrelation between the security’s prices.<sup>28</sup> Our results will shed light on whether litigants and the courts are presenting evidence consistent with the results of finance theory and empirics.

## II. ARBITRAGE RISK AS A NEGATIVE PROXY FOR MARKET EFFICIENCY

Capital market efficiency describes how completely and

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27. *Id.* at 2421.

28. See *In re DVI, Inc. Sec. Litig.*, 639 F.3d 623 (3d Cir. 2011), *abrogated in part by* *Amgen Inc. v. Conn. Ret. Plans & Trust Funds*, 133 S. Ct. 1184 (2013); *Teamsters Local 445 Freight Div. Pension Fund v. Bombardier, Inc.*, 546 F.3d 196, 204 n.11 (2d Cir. 2008); *In re Xcelera.com Sec. Litig.*, 430 F.3d 503, 508–09 (1st Cir. 2005); *In re Polymedica Corp. Sec. Litig.*, 432 F.3d 1, 18 n.21 (1st Cir. 2005); *Unger v. Amedisys Inc.*, 401 F.3d 316, 323 (5th Cir. 2005); *Gariety v. Grant Thornton, LLP*, 368 F.3d 356, 368 (4th Cir. 2004); *Binder v. Gillespie*, 184 F. 3d 1059, 1064–65 (9th Cir. 1999); *Krogman v. Sterritt*, 202 F.R.D. 467, 478 (N.D. Tex. 2001).

accurately the pricing signal works. When all publicly available information is reflected in a security's price, the market for the security is said to be semistrong form efficient.<sup>29</sup> We interpret the concept not as an either/or, binary construct, but rather as a relative concept occurring along a continuum, and thus one often refers to a market's relative efficiency.<sup>30</sup> The pricing signal is thought to work through the actions of all traders who, whatever their level of knowledge and sophistication, convey their individual valuations to the market through their buy and sell decisions at various price points.<sup>31</sup> The collective actions of all traders thus push the price of a particular security toward its market equilibrium level, but do not necessarily take the market for the security all the way to perfect semistrong form efficiency.

Arbitrageurs are investors who trade on information about relative values. They trade investments that are or should be fundamentally correlated but for which they believe the market valuations are deviating from the fundamental relation. Thus arbitrageurs attempt to take advantage of the market pricing discrepancies between otherwise fundamentally correlated securities in order to earn trading profits. This activity of exploiting situations in which markets are not efficient assists the pricing signal by conveying information to the market and helping to push the market to efficiency, but does not necessarily take the market all the way to perfect semistrong form efficiency.

But as Pontiff explains, because of "costly arbitrage," arbitrageurs are unlikely to ever completely eliminate mispricing.<sup>32</sup> He identifies two sources of arbitrage costs: transactions costs (*e.g.*, commissions, brokerage fees) and holding costs (*e.g.*, opportunity cost of capital and the idiosyncratic risk of a security), and he stresses the importance of idiosyncratic risk in making arbitrage a costly endeavor.<sup>33</sup> As arbitrageurs construct their hedge portfolios of

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29. See Eugene Fama, *Efficient Capital Markets: A Review of Theory and Empirical Work*, 25 J. FINANCE 383, 404 (1970); BURTON G. MALKIEL, *Efficient Market Hypothesis*, in NEW PALGRAVE DICTIONARY OF MONEY AND FINANCE 322–36 (P. Newman, M. Milgate, & J. Eatwell eds., 1st ed. 1987).

30. See, *e.g.*, John Campbell, Andrew Lo, and A. Craig MacKinlay, *The Econometrics of Financial Markets*, Princeton University Press, Princeton, NJ (1997).

31. See, *e.g.*, HARRIS, *supra* note 6, at chapters 10 and 17.

32. Pontiff, *supra* note 7, at 39–40.

33. *Id.* at 37, 41.

investments, supposedly correlated in returns, they cannot find perfectly positive correlations in returns and thus perfect substitutes as investments (or perfectly negative correlations in returns and perfect complements as investments), so they are always exposed to the vagaries of each individual security they hold. Even aggregated across a number of investment positions within the hedge portfolio, the legs of the arbitrage, arbitrageurs cannot eliminate the idiosyncratic risk of any security.

As a result of the costs of arbitrage, including idiosyncratic risk, market inefficiencies will always remain; the better arbitrage works, the more efficient the market for a security is likely to be. As a result, the costs of arbitrage for a security provide a means to test the efficiency of the security.

Pontiff concludes that “idiosyncratic risk is the single largest cost faced by arbitrageurs,” and that “idiosyncratic risk is the single largest barrier to arbitrage.”<sup>34</sup> Our notion of arbitrage risk is a generalization of the standard notion of idiosyncratic risk, defined as the standard deviation of residuals from the Capital Asset Pricing Model (CAPM).<sup>35</sup> If an arbitrageur is constrained to having access to only the market index and a risk-free instrument to devise an optimal zero-net-investment arbitrage portfolio, then the risk of the resulting optimal arbitrage portfolio is the standard idiosyncratic risk. For our notion of *ex ante* or *ex post* arbitrage risk, however, we model the arbitrageur as optimally choosing the components of the arbitrage portfolio from the universe of the market index *and* all exchange-listed U.S. common stocks, based on financial data on the returns of the stock of interest, all candidate securities, and the risk-free instrument over the measurement period. Then, given the optimal choice of the components of the arbitrage portfolio, and the risk-free instrument, we model the arbitrageur as choosing the optimal hedge ratios under a zero-net-investment constraint. The risk of this optimal arbitrage portfolio is the arbitrage risk of the stock, our negative proxy for market efficiency. Our interviews with traders “in the trenches” confirm this overall structure of

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34. *Id.*

35. *See, e.g.,* William F. Sharpe, Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk, 19 *J. FINANCE* 425 (1964); Harry Markowitz, *The Early History of Portfolio Theory: 1600–1960*, 55 *FIN. ANALYSTS J.* 5 (1999); Merton H. Miller, *The History of Finance*, *J. PORTFOLIO MGMT.*, 95 (1999).

hedging behavior by arbitrageurs. Wurgler and Zhuravskaya construct a similar notion of arbitrage risk, but their substitute securities are selected from similarities in product market characteristics.<sup>36</sup>

We thus develop a practical and general (negative) proxy for market efficiency by quantifying the arbitrage risk associated with each individual security. This simple (negative) proxy for market efficiency then allows for an examination of a variety of factors that have been proffered by economists and lawyers as affecting market efficiency.

We calculate the arbitrage risk of a stock over a defined relevant period as follows:

We select the relevant period, which can be a day, a week, a court-determined “class” period, or other time frame of relevance (simply for presentation in this paper, we have chosen annual periods).

We choose a period immediately prior to the relevant period as the measurement period, because the arbitrageur would not have had access to relevant-period data at the beginning of the relevant period (also for simplicity, we have chosen the year prior to the relevant period as the measurement period).

On the basis of the measurement period, we determine the lowest risk portfolio that an arbitrageur would use to benefit from mispricing, as explained above. We call this portfolio the arbitrage portfolio for the stock.

The *ex ante* (or baseline) arbitrage risk of the security for the relevant period is the risk of the arbitrage portfolio over the relevant period.

If it is not possible (or desirable) to use a different measurement period—e.g., if the period of interest immediately follows an initial public offering—then we can do the calculations where the measurement period is the same as the relevant period, and we call this the *ex post* arbitrage risk of the security for the relevant period. For this paper, we focus on *ex ante* (baseline) arbitrage risk, but we do sensitivity analyses with *ex post* arbitrage risk as another negative proxy for market efficiency.

Consider a stock that is underpriced (overpriced) according to the information available to an arbitrageur. In

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36. Jeffrey Wurgler and Ekaterina Zhuravskaya, *Does Arbitrage Flatten Demand Curves for Stocks*, 75 J. BUS. 583, 592–93 (2002).

order to exploit this profitable opportunity, the arbitrageur will construct the following arbitrage portfolio with zero net investment, thereby reaping arbitrage profits when closing out the investment portfolio:

Arbitrage numerator: go long (short) on the mispriced stock; let's say by \$1 (purely a normalization).

Other arbitrage legs:

Go short (long) on  $N$  other securities. We use  $N = 5$  legs for the baseline model, but the results do not change qualitatively with values of  $N = 10$  or  $20$ .

Go short (long) on the risk-free asset.

The total amount on these legs has to add up to \$1 short (long).

How much to go short (long) on each leg is called the corresponding Hedge Ratio.<sup>37</sup>

The risk of a portfolio over a period is defined as the standard deviation of daily returns of the portfolio over that period. Since the arbitrageur does not have access to all relevant period data at the beginning of the relevant period, we choose a period immediately prior to the relevant period as the measurement period, and the arbitrage portfolio is selected to minimize the risk (i.e., the standard deviation of daily returns) of the arbitrage portfolio<sup>38</sup> over the measurement period, with the requirement described above that the arbitrage portfolio require zero investment. The risk of the arbitrage portfolio over the relevant period is the *arbitrage risk of the stock for the relevant period*. For this paper:

We measure arbitrage risk for U.S. exchange-listed common stocks,

We consider only U.S. exchange-listed common stocks, and the S&P 500 index, as candidate legs of the hedge portfolio, and

We use daily returns on six-month U.S. Treasury bills as the daily risk-free rate.<sup>39</sup>

Table 2 shows examples of exchange-listed stocks and their estimated market efficiency percentiles in 2010, on the

37. *Id.* at 593 (providing the computational simplification used here).

38. "Arbitrageurs generally construct hedge portfolios to minimize the total risk of the portfolio." HARRIS, *supra* note 6, at 348.

39. BD. OF GOVERNORS OF THE FED. RESERVE SYS., *Selected Interest Rates (Daily)*, FEDERAL RESERVE STATISTICAL RELEASE H.15, <http://www.federalreserve.gov/releases/h15/data.htm> (last updated Feb. 18, 2015).

basis of our measure of arbitrage risks of all U.S. exchange-listed common stocks.

*Table 2: Examples of Market Efficiency Percentiles in 2010*

<b>Ticker</b>	<b>Company</b>	<b>Market Efficiency Percentile</b>
ADP	AUTOMATIC DATA PROCESSING INC	100
BRK	BERKSHIRE HATHAWAY INC DEL	100
DUK	DUKE ENERGY CORP NEW	100
ED	CONSOLIDATED EDISON INC	100
HNZ	HEINZ H J CO	100
JNJ	JOHNSON & JOHNSON	100
JW	WILEY JOHN & SONS INC	100
MO	ALTRIA GROUP INC	100
NST	NSTAR	100
ADM	ARCHER DANIELS MIDLAND CO	91
CAT	CATERPILLAR INC	91
CSCO	CISCO SYSTEMS INC	91
EMC	E M C CORP MA	91
BLK	BLACKROCK INC	81
CBT	CABOT CORP	81
ENDP	ENDO PHARMACEUTICALS HLDNGS INC	81
GPS	GAP INC	81
AMZN	AMAZON COM INC	80
AXP	AMERICAN EXPRESS CO	80
ACF	AMERICREDIT CORP	51
CIEN	CIENA CORP	51
COBZ	COBIZ FINANCIAL INC	51
CPHD	CEPHEID	51
DAL	DELTA AIR LINES INC	51
ACAT	ARCTIC CAT INC	26
APKT	ACME PACKET INC	26
ARTW	ARTS WAY MANUFACTURING INC	26
ATAC	A T C TECHNOLOGY CORP	26
AVII	A V I BIOPHARMA INC	26
TEAR	OCCULOGIX INC	1



TLX	TRANS LUX CORP	1
TSTR	TERRESTAR CORP	1
VNDA	VANDA PHARMACEUTICALS INC	1
ZANE	ZANETT INC	1

Table 3 demonstrates that different stocks display different patterns of market efficiency across time.

*Table 3: Examples of Market Efficiency Percentiles Over Time*

<b>Ticker</b>	<b>Company Name</b>	<b>Year</b>	<b>Market Efficiency Percentile</b>
ALG	ALAMO GROUP INC	2001	96
ALG	ALAMO GROUP INC	2002	83
ALG	ALAMO GROUP INC	2003	95
ALG	ALAMO GROUP INC	2004	68
ALG	ALAMO GROUP INC	2005	46
ALG	ALAMO GROUP INC	2006	54
ALG	ALAMO GROUP INC	2007	60
ALG	ALAMO GROUP INC	2008	68
ALG	ALAMO GROUP INC	2009	47
ALG	ALAMO GROUP INC	2010	58
CNLG	CONOLOG CORP	2001	2
CNLG	CONOLOG CORP	2002	3
CNLG	CONOLOG CORP	2003	3
CNLG	CONOLOG CORP	2004	1
CNLG	CONOLOG CORP	2005	2
CNLG	CONOLOG CORP	2006	1
CNLG	CONOLOG CORP	2007	1
CNLG	CONOLOG CORP	2008	1
CNLG	CONOLOG CORP	2009	3
CNLG	CONOLOG CORP	2010	2
ED	CONSOLIDATED EDISON INC	2001	100

ED	CONSOLIDATED EDISON INC	2002	98
ED	CONSOLIDATED EDISON INC	2003	100
ED	CONSOLIDATED EDISON INC	2004	100
ED	CONSOLIDATED EDISON INC	2005	100
ED	CONSOLIDATED EDISON INC	2006	100
ED	CONSOLIDATED EDISON INC	2007	100
ED	CONSOLIDATED EDISON INC	2008	100
ED	CONSOLIDATED EDISON INC	2009	100
ED	CONSOLIDATED EDISON INC	2010	100
FRE	FEDERAL HOME LOAN MORTGAGE CORP	2001	100
FRE	FEDERAL HOME LOAN MORTGAGE CORP	2002	100
FRE	FEDERAL HOME LOAN MORTGAGE CORP	2003	91
FRE	FEDERAL HOME LOAN MORTGAGE CORP	2004	97
FRE	FEDERAL HOME LOAN MORTGAGE CORP	2005	94
FRE	FEDERAL HOME LOAN MORTGAGE CORP	2006	95
FRE	FEDERAL HOME LOAN MORTGAGE CORP	2007	40
FRE	FEDERAL HOME LOAN MORTGAGE CORP	2008	3
FRE	FEDERAL HOME LOAN MORTGAGE CORP	2009	3
MSFT	MICROSOFT CORP	2001	55
MSFT	MICROSOFT CORP	2002	89
MSFT	MICROSOFT CORP	2003	92
MSFT	MICROSOFT CORP	2004	96
MSFT	MICROSOFT CORP	2005	98
MSFT	MICROSOFT CORP	2006	87
MSFT	MICROSOFT CORP	2007	95
MSFT	MICROSOFT CORP	2008	95
MSFT	MICROSOFT CORP	2009	89
MSFT	MICROSOFT CORP	2010	95

### III. RELATION OF ARBITRAGE RISK TO STANDARD FACTORS: EMPIRICAL FINDINGS

We test the empirical relation with arbitrage risk (as a negative proxy for market efficiency) of factors relied upon by courts and others as determinants of market efficiency.<sup>40</sup> This is done for all U.S. exchange-listed common stocks from 1988 (the year of the landmark Basic decision detailed in Part I) to 2010, the last year for which we have full data, with the following restrictions.

We restricted attention to stock-year combinations consisting of stocks that had one PERMNO, one ticker, and one CUSIP over the year in the Center for Research in Security Prices (CRSP) Daily Stock Database. We utilized each stock available from CRSP,<sup>41</sup> for which the following data exist for at least 75% of trading days during each of the relevant and measurement periods:

- Returns
- Shares outstanding
- Trading volume
- Closing bid
- Closing ask
- Exchange membership
- Number of market makers.

To include a particular stock in our analysis, we also required data to be available for:

- The number of securities analysts (from I/B/E/S)
- Insider holdings (from Thomson Reuters)
- Institutional holdings (from Thomson Reuters)
- Inclusion in (or exclusion from) the Dow Jones Industrial Average (DJIA) index (from Phyllis Pierce, *The Dow Jones Averages 1885–1995*).

Appendix 1 shows the number of stocks at the various stages after imposing the restrictions described above.

From this data, we develop measures for the various

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40. See generally Brad Barber, Paul A. Griffin & Baruch Lev, *The Fraud-on-the-Market Theory and the Indicators of Common Stocks' Efficiency*, J. CORP. L. 285 (1994) (using a different proxy).

41. shrcd = 10 or 11 in the Daily Stock Database from the Center for Research in Security Prices.

factors that courts and others have relied upon as influencing market efficiency. These measures are each defined below.

- Turnover: mean daily turnover (volume)/(shares outstanding) over the relevant period.
- Number of security analysts: number of security analysts who announced at least one projection about the security during the relevant period.
- Number of market makers for Nasdaq stocks: highest, over the relevant period, of the number of market makers on each day.
- Market capitalization: mean of daily logarithm of market capitalization over the relevant year.
- Bid-ask spread: mean of daily relative spread (closing ask – closing bid)/(closing price) over the relevant period.
- Public float ratio: mean of quarterly public float ratio (shares outstanding – insider holdings)/(shares outstanding) over the relevant period.
- Institutional ownership ratio: institutional ownership ratio (institutional holdings)/(shares outstanding) over the relevant period.
- Serial correlation in CAPM: By performing the Durbin-Watson Test on the CAPM on daily returns for a stock for a calendar year, we obtain the p-value for positive serial correlation and the p-value for negative serial correlation.<sup>42</sup> The negative of the minimum of these p-values is a positive measure of serial correlation in the CAPM for the particular stock for the relevant year.
- Explanatory power of CAPM:  $R^2$  of the CAPM for the security in the relevant period.
- Inclusion in the DJIA: If a stock is on the DJIA for an entire year, the indicator variable for that stock for that year is one. If a stock is not on the DJIA for an entire year, the indicator variable for that stock for that year is zero. If a stock is on the DJIA for only part of the year, the observation for that stock for that year is deleted from the regression.

Summary statistics for arbitrage risk and for each of the explanatory variables are shown in Appendix 2.

For this paper, we perform a reduced-form regression of

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42. See, e.g., WILLIAM H. GREENE, *ECONOMETRIC ANALYSIS* (4th ed. 2000).

arbitrage risk<sup>43</sup> on various factors of market efficiency, controlling for year. The results of this regression—which uses 35,587 observations (stock-years) and has  $R^2 = 0.463$  and adjusted  $R^2 = 0.462$  (see Appendix 3)—are shown in Table 4.

Table 4: Detailed Regression Results

Dependent Variable: Arbitrage Risk							
Factor	Coefficient	Under Homoscedasticity			Heteroscedasticity-Robust		
		Standard Error	t-Statistic	p-Value	Standard Error	t-Statistic	p-Value
<i>Cammer v. Bloom</i>							
Turnover	74.366%	3.881%	19.160	<0.0001	6.286%	11.830	<0.0001
Number of Analysts	0.006%	0.004%	1.770	0.077	0.002%	3.100	0.0019
Number of Nasdaq Market Makers	0.019%	0.003%	5.780	<0.0001	0.004%	4.880	<0.0001
<i>Unger v. Amidesys</i>							
Market Capitalization	-0.406%	0.032%	-12.880	<0.0001	0.043%	-9.360	<0.0001
Bid-Ask Spread	48.975%	1.750%	27.980	<0.0001	9.077%	5.400	<0.0001
Public Float Ratio	-0.821%	0.179%	-4.600	<0.0001	0.880%	-0.930	0.3508
<i>Other</i>							
Institutional Ownership Ratio	-0.629%	0.138%	-4.560	<0.0001	0.142%	-4.420	<0.0001
Serial Correlation	-0.423%	0.195%	-2.170	0.03	0.148%	-2.860	0.0042
Explanatory Power	-1.411%	0.280%	-5.040	<0.0001	0.166%	-8.520	<0.0001
Inclusion in Dow Jones Index	1.030%	0.295%	3.490	0.0005	0.136%	7.550	<0.0001

43. It is worth noting again that, *ceteris paribus*, market efficiency has a negative relation with arbitrage risk.

Listed on NYSE	0.467%	0.145%	3.220	0.0013	0.153%	3.040	0.0024
Listed on AMEX	-0.351%	0.163%	-2.150	0.0314	0.112%	-3.150	0.0016
Listed on Other Exchange	0.800%	2.255%	0.350	0.7227	0.640%	1.250	0.2113
Year 1988	6.788%	0.571%	11.890	<0.0001	0.878%	7.730	<0.0001
Year 1989	7.640%	0.550%	13.900	<0.0001	0.872%	8.760	<0.0001
Year 1990	12.130%	0.563%	21.550	<0.0001	1.267%	9.570	<0.0001
Year 1991	45.566%	0.568%	80.230	<0.0001	4.618%	9.870	<0.0001
Year 1992	11.421%	0.464%	24.610	<0.0001	0.976%	11.700	<0.0001
Year 1993	6.968%	0.457%	15.240	<0.0001	0.850%	8.190	<0.0001
Year 1994	6.826%	0.449%	15.210	<0.0001	0.852%	8.010	<0.0001
Year 1995	6.743%	0.447%	15.070	<0.0001	0.853%	7.910	<0.0001
Year 1996	7.066%	0.444%	15.920	<0.0001	0.856%	8.260	<0.0001
Year 1997	7.381%	0.439%	16.830	<0.0001	0.852%	8.670	<0.0001
Year 1998	7.968%	0.433%	18.420	<0.0001	0.848%	9.400	<0.0001
Year 1999	7.966%	0.437%	18.210	<0.0001	0.856%	9.300	<0.0001
Year 2000	8.747%	0.436%	20.070	<0.0001	0.859%	10.180	<0.0001
Year 2001	8.742%	0.424%	20.610	<0.0001	0.840%	10.400	<0.0001
Year 2002	8.539%	0.414%	20.620	<0.0001	0.830%	10.290	<0.0001
Year 2003	7.963%	0.412%	19.330	<0.0001	0.819%	9.730	<0.0001
Year 2004	7.801%	0.417%	18.730	<0.0001	0.823%	9.480	<0.0001
Year 2005	7.694%	0.416%	18.480	<0.0001	0.823%	9.340	<0.0001
Year 2006	7.684%	0.415%	18.510	<0.0001	0.824%	9.320	<0.0001
Year 2007	8.080%	0.411%	19.650	<0.0001	0.825%	9.800	<0.0001
Year 2008	9.465%	0.399%	23.710	<0.0001	0.828%	11.430	<0.0001
Year 2009	9.677%	0.393%	24.630	<0.0001	0.823%	11.760	<0.0001
Year 2010	7.928%	0.396%	20.010	<0.0001	0.820%	9.670	<0.0001

As with the results summarized in Table 1, we checked the sensitivity of these results through a number of additional analyses. For one set, we replaced turnover with logarithm of volume (or logarithm of dollar volume) but removed market capitalization from the list of factors, reflecting the fact that, *ceteris paribus*, the volume for a stock with higher market capitalization will be higher. For this set, we found that the results were the same as in Table 4, except that arbitrage risk

was negatively and significantly affected by number of analysts; negatively but insignificantly affected by number of market makers (for Nasdaq stocks); negatively but ambiguously affected by serial correlation; and negatively and significantly affected by inclusion in the DJIA (the latter makes sense because in this set, market capitalization is not used as an explanatory factor, whereas it was used as such for the results in Table 4). The second set uses only the *Cammer* factors as explanatory variables. For this set, we found that the results were the same as in Table 4, except that arbitrage risk is negatively but insignificantly affected by logarithm of volume (or logarithm of dollar volume); and negatively and significantly affected by number of analysts.

#### A. *Trading Volume*

The positive relation of trading volume or turnover with market efficiency is often considered almost axiomatic—statements such as “the more thinly traded the stock, the less efficient the market is” are common. However, this is not necessarily the case.<sup>44</sup> In order to appreciate how trading volume (or turnover) is often misunderstood, we need to appreciate that:

[I]nvestors trade among themselves because they are different. . . . The difference in their response to the same information generates trading. The greater the information asymmetry, the larger the abnormal trading volume when public news arrives.<sup>45</sup>

We would observe higher trading volume or turnover in the market for an asset if and only if:<sup>46</sup>

- 1) Transaction costs (including bid-ask spread, commissions, and search costs) are low relative to

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44. See also MAUREEN O'HARA, *MARKET MICROSTRUCTURE THEORY* (2nd ed. 1997); Lawrence Blume, David Easley & Maureen O'Hara, *Market Statistics and Technical Analysis: The Role of Volume*, 49 J. FIN. 153 (1994); Jonathan Karpoff, *The Relation Between Price Changes and Trading Volume: A Survey*, 22 J. FIN. & QUANTITATIVE ANALYSIS 109 (1987); Erik R. Sirri, Edith S. Hotchkiss & Michael A. Goldstein, *Transparency and Liquidity: A Controlled Experiment on Corporate Bonds*, 20 REV. FIN. STUDIES 235 (2007); and Scott E. Stickel & Robert E. Verrecchia, *Evidence that Trading Volume Sustains Stock Price Changes*, 50 FIN. ANALYSTS J. 57 (1994).

45. Jiang Wang, *A Model of Competitive Stock Trading Volume*, 102 J. POL. ECON. 127, 128–29 (1994).

46. See Rajeev Bhattacharya, *Structural Models of Market Efficiency*, Mimeo (2012), for detailed derivations.

dispersion in investor valuations.

- 2) Low-valuers hold more securities than high-valuers, and short sales costs are low relative to dispersion in investor valuations.

Higher trading volume will therefore be observed with:

- Lower transaction costs,
- Lower short sales costs,
- Higher dispersion of investor valuations, and/or
- Higher likelihood that an investor holds the asset.

Clearly, market efficiency would be facilitated by lower transaction costs and lower short sales costs—and when a claim is made about higher trading volume being a factor favoring market efficiency, it is often implicitly based on this argument. However, there is no reason to conclude that, *ceteris paribus*, a higher dispersion in investor valuations, or a higher likelihood that an investor holds the asset, would lead to higher market efficiency.<sup>47</sup>

Given these contrary dependencies, the relation of market efficiency with turnover is fundamentally an empirical question, and our empirical finding, for all U.S. exchange-listed common stocks from 1988 to 2010,<sup>48</sup> is that turnover negatively and significantly affects market efficiency.

### *B. The Number of Market Makers*

From the above discussion, we can see that the demand for market making services in a Nasdaq stock is an increasing function of dispersion in investor valuations—*i.e.*, everything else remaining the same, a firm is more likely to be a market maker in a Nasdaq stock if the investor valuation profile for that stock is more dispersed, because there would be more trades to enable more profits.

However, the higher the number of market makers, the higher the competition for trades would be, and this would reduce the transaction costs of trades. Therefore, a higher number of market makers would be observed with higher dispersion (not unambiguously facilitating market efficiency, as shown above) and/or lower transaction cost (facilitating market efficiency).

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47. With high dispersion in investor valuations, we can have a high trading volume (much of which can be due to noise trading), and a higher dispersion in investor valuations does not necessarily add to the efficiency of a market.

48. Restrictions are detailed at the beginning of Section 3.



Therefore, as above, the relation between the number of Nasdaq market makers and market efficiency is fundamentally empirical, and our empirical finding, for all U.S. exchange-listed common stocks from 1988 to 2010,<sup>49</sup> is that arbitrage risk for a Nasdaq stock is positively and significantly related to the number of market makers for that stock; and that market efficiency for a Nasdaq stock is negatively and significantly related to the number of market makers for that stock.

### *C. Serial Correlation*

Serial correlation in the CAPM of a stock's daily returns, suggesting a day-to-day pattern of repeated performance, and is often considered evidence of market inefficiencies. Some scholars have suggested other explanations<sup>50</sup> of serial correlation that are consistent with market efficiency. In the presence of such inconsistent findings, this dependence becomes an empirical question. Our empirical findings suggest that, on the basis of all U.S. exchange-listed common stocks from 1988 to 2010,<sup>51</sup> serial correlation affects market efficiency positively and significantly.

### *D. Other Factors*

The number of securities analysts is a *Cammer* factor, and other courts have suggested using public float as a factor favorable to a finding of market efficiency. Our empirical findings suggest that, on the basis of all U.S. exchange-listed common stocks from 1988 to 2010,<sup>52</sup> the number of securities analysts and public float have an ambiguous impact on market efficiency.

Consistent with the position taken by courts and others, we find that market capitalization and institutional ownership have significant empirical support based on our analysis. The bid-ask spread of a stock, which is a measure of transaction cost of a stock, has been used by courts as a factor inhibiting market efficiency, and our results confirm this position, too.

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49. *Id.*

50. Ray Ball & S.P. Kothari, *Nonstationary Expected Returns: Implications for Tests of Market Efficiency and Serial Correlation in Returns*, 25 J. FIN. ECON. 51 (1989).

51. Restrictions are detailed at the beginning of Section 3.

52. *Id.*

Although we have not found a court ruling where the inclusion of the relevant stock in a major index is a factor favorable to a finding of market efficiency, expert reports and testimony commonly use such an argument. It is interesting to note, however, that because we control for market capitalization, what the indicator variable corresponding to membership in the DJIA measures is purely how much the inclusion of the stock in the DJIA adds to the efficiency of the market for the stock, over and above what the stock's market capitalization predicts. We find, for all U.S. exchange-listed common stocks from 1988 to 2010,<sup>53</sup> that inclusion in the DJIA, over and above its size, significantly reduces the efficiency of the market for its stock.

It should be noted that these results are robust to alterations in the statistical model. We performed sensitivity analyses with a) *ex post* arbitrage risk, b) sign constraints on the correlations of returns in the determination of legs of the arbitrage portfolio, c) sign constraints on the hedge ratios, d) number of legs in the arbitrage portfolio, and e) the risk-free rate. The empirical findings are qualitatively the same as for the baseline regressions.

#### CONCLUSIONS

We discuss arbitrage risk, a negative proxy for market efficiency based on financial economics, and show how it can be applied to securities class actions.

We also test the dependence of arbitrage risk on various standard factors used by courts in the determination of market efficiency. Some of these empirical findings do not seem consistent with “conventional wisdom”—as Warren Buffett is famously supposed to have said, “Well, it may be all right in practice, but it will never work in theory.” However, we suggest that our empirical findings are actually consistent with economic theory.

The list of determinants of market efficiency considered in this paper is not exhaustive. Academic research and interviews with financial practitioners have suggested additional factors such as external and internal regulations, and industrial organization (including supply and demand structures) of product markets. We intend to utilize data on

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53. *Id.*

industry codes, and industry properties such as pricing and concentration, to analyze these additional factors affecting market efficiency.

*Cammer* Factor (5)—the cause-and-effect relationship between material disclosures and changes in the security's price—is typically analyzed using event studies, which are joint tests of market efficiency and significance of particular events.<sup>54</sup> Using summaries of such event studies as proxies for market efficiency in the context of a macro-analysis like ours would be possible only with substantial simplifications/assumptions such as restricting our attention to earnings announcements different from consensus analyst forecasts as the only surprises for a stock. We intend to undertake such an analysis in the future.

Finally, we propose to investigate alternative proxies for market efficiency such as put-call parity of options on the security of interest in order to test the robustness of our results across varying measures of market efficiency.

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54. See, e.g., CAMPBELL, ET AL., *supra* note 30; and Pontiff, *supra* note 7, at 35.

<i>Appendix 1: Number of Unique Stock-Year Combinations</i>				
<b>Year</b>	<b>All</b>	<b>With Unique PERMNO, Ticker, and CUSIP in Calendar Year</b>	<b>With Calendar Year Trading Restrictions</b>	<b>After Mergers with Other Data</b>
<b>All Years</b>	141,591	131,779	106,585	37,111
<b>1988</b>	5,323	4,839	4,040	538
<b>1989</b>	4,980	4,633	4,053	578
<b>1990</b>	4,772	4,454	3,923	585
<b>1991</b>	4,843	4,523	3,901	610
<b>1992</b>	6,434	5,973	4,669	652
<b>1993</b>	6,976	6,450	4,911	754
<b>1994</b>	7,367	6,871	5,160	866
<b>1995</b>	7,757	7,170	5,507	990
<b>1996</b>	8,282	7,606	5,605	1,098
<b>1997</b>	8,468	7,710	5,781	1,237
<b>1998</b>	8,254	7,544	5,869	1,415
<b>1999</b>	7,917	7,218	5,551	1,542
<b>2000</b>	7,528	6,835	5,198	1,646
<b>2001</b>	6,692	6,252	5,016	1,765
<b>2002</b>	5,955	5,562	4,850	1,917
<b>2003</b>	5,497	5,231	4,535	2,034
<b>2004</b>	5,286	5,032	4,314	2,185
<b>2005</b>	5,228	4,976	4,162	2,379
<b>2006</b>	5,179	4,893	4,085	2,619
<b>2007</b>	5,155	4,891	3,993	2,859
<b>2008</b>	4,843	4,591	3,923	2,900
<b>2009</b>	4,533	4,343	3,871	3,040
<b>2010</b>	4,322	4,182	3,668	2,902

Appendix 2: Summary Statistics

Year	Arbitrage Risk (percentage)		Turnover (percentage)		Number of Analysts		Number of Nasdaq Market Makers		Log of Market Capitalization		Bid-Ask Spread (percentage)		Public Float Ratio (percentage)		Institutional Ownership Ratio (percentage)		Serial Correlation (percentage)		Explanatory Power (percentage)	
	Mean	Std. Dev	Mean	Std. Dev	Mean	Std. Dev	Mean	Std. Dev	Mean	Std. Dev	Mean	Std. Dev	Mean	Std. Dev	Mean	Std. Dev	Mean	Std. Dev	Mean	Std. Dev
All	3.97	9.28	0.75	0.86	11.8	11.3	16.3	20	13.21	1.87	1.37	2.30	98.01	16.27	56.65	29.18	-16.83	15.71	16.96	16.23
1988	2.85	1.74	0.28	0.33	13.2	12.2	5.2	9.2	12.63	1.8	4.22	4.49	97.43	12.97	37.92	19.18	-14.52	15.23	14.62	16.00
1989	3.49	3.11	0.30	0.30	14.3	13.1	5.9	10	12.66	1.89	3.76	3.10	97.73	8.51	38.35	20.11	-13.87	15.34	10.64	12.95
1990	7.21	10.73	0.28	0.27	11.9	10.6	5.6	9.8	12.6	1.95	5.46	5.31	98.33	8.04	39.58	20.82	-13.08	15.09	12.94	13.55
1991	39.66	58.20	0.34	0.37	10.7	9.7	5.4	9.6	12.72	1.93	6.05	8.49	96.51	18.65	42.78	21.64	-15.56	15.65	10.48	10.70
1992	7.25	11.99	0.35	0.36	10.7	9.8	5.9	10.1	12.92	1.89	2.99	4.85	95.05	29.43	44.97	22.45	-14.36	15.13	6.27	7.30
1993	2.73	1.60	0.40	0.46	11.8	11.1	6.4	10.2	12.91	1.86	2.75	2.53	96.27	35.65	44.81	22.55	-14.34	15.47	4.59	5.46
1994	2.78	1.81	0.40	0.50	12.4	12.4	6.9	10.3	12.8	1.89	3.10	3.42	95.18	41.80	44.38	23.79	-13.73	15.73	6.49	7.00
1995	2.83	2.00	0.49	0.61	12.5	12.5	7	10.1	12.8	1.91	3.17	3.71	94.73	41.66	44.14	24.87	-12.93	14.86	3.74	4.58
1996	2.91	1.76	0.51	0.57	12.2	12.6	6.8	9.3	12.9	1.89	2.86	2.47	96.86	13.73	43.99	24.86	-14.27	15.22	6.39	8.12
1997	3.01	1.76	0.54	0.66	12	12.4	7.6	10.1	12.99	1.9	2.56	2.51	97.31	15.35	45.83	25.92	-14.07	15.13	9.13	11.01
1998	3.49	2.08	0.52	0.58	11.4	11.8	9.8	12.3	13.03	1.91	2.39	2.07	97.01	35.48	46.61	25.34	-15.30	15.76	11.26	10.61
1999	3.72	2.00	0.57	0.71	11.8	12	10.7	13.7	13.01	1.94	2.52	2.52	97.84	25.76	47.60	26.70	-15.68	15.57	4.27	6.65
2000	4.45	2.37	0.71	0.87	11.7	12	12.1	15.8	13.15	1.97	2.38	2.44	98.54	6.24	48.23	27.05	-15.17	15.50	7.31	7.61
2001	3.93	2.36	0.66	0.79	11.6	11.4	14.3	18.2	13.22	1.92	1.58	1.65	98.62	4.77	51.89	27.61	-17.11	15.98	13.09	12.36
2002	3.54	2.08	0.68	0.80	11.4	11.2	16.5	20	13.19	1.86	1.28	1.39	98.44	8.64	54.16	27.61	-15.80	15.54	19.15	15.49
2003	2.73	1.67	0.73	0.85	11.7	11.8	16.6	20.2	13.28	1.8	0.71	0.88	97.76	15.11	56.73	28.18	-18.39	15.86	17.12	14.60
2004	2.37	1.30	0.80	1.24	11.3	10.7	18.3	21.1	13.51	1.72	0.41	0.59	98.33	5.62	61.43	28.61	-17.04	15.59	16.15	11.30
2005	2.27	1.23	0.80	0.93	11.2	10.4	20.3	22.4	13.53	1.76	0.39	0.58	98.37	9.15	63.50	29.50	-18.80	15.59	15.63	12.39
2006	2.24	1.17	0.86	0.77	11.6	10.4	21.4	22.5	13.58	1.75	0.31	0.45	98.47	6.43	65.50	29.18	-19.33	15.53	16.16	12.18
2007	2.70	1.45	1.00	0.90	11.6	10.2	21.8	21.8	13.58	1.78	0.38	0.59	98.50	5.43	69.55	30.44	-18.65	15.81	19.83	14.94
2008	4.45	2.68	1.13	0.96	11.7	10.5	22.6	22.1	13.58	1.85	0.95	1.71	98.81	4.63	67.85	30.03	-16.42	15.65	30.29	19.04
2009	4.77	2.70	1.02	0.98	11.6	11.1	23.3	22.3	13	1.88	0.97	1.82	98.78	5.28	62.50	28.80	-18.04	15.87	29.39	18.87
2010	2.57	1.49	0.95	0.91	12.9	12	24	23.4	13.37	1.85	0.47	0.89	98.99	3.70	64.93	28.74	-19.53	15.47	30.44	18.50

*Appendix 3: Summary Regression Results*

<b>Number of Observations Read</b>						37,111
<b>Number of Observations Used</b>						35,587
<b>Number of Observations with Missing Values</b>						1,524
		<b>Degrees of Freedom</b>	<b>Sum of Squares (SS)</b>	<b>Mean SS</b>	<b>F-Statistic</b>	<b>p-Value</b>
<b>Model</b>		36	93.1478	2.58744	851.3	<0.0001
<b>Error</b>		35,551	108.0536	0.00304		
<b>Uncorrected Total</b>		35,587	201.2014			
<b>R<sup>2</sup></b>		0.463				
<b>Adjusted R<sup>2</sup></b>		0.462				